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ABSTRACT

In the attempt to improve the quality of geometry instruction in schools, researchers and teachers alike have given considerable attention to the van Hiele theory of geometry learning and development, which proposes a series of cognitive levels through which every geometry student passes. This paper reports a study to determine the extent to which factors of age, gender, grade point average, standardized achievement test scores, and geometry achievement acted as predictors for van Hiele levels in two groups of 8th through 11th grade students (N=328): entering geometry (i.e., algebra students) and exiting geometry students. Individual variables determined to be significantly correlated with van Hiele levels among all subjects tested were class level (geometry or algebra), standardized achievement test scores, and geometry achievement test scores. For the geometry subgroup, significant variables were standardized achievement test scores, geometry achievement test scores, and age (negative correlation). For the algebra subgroup, only the geometry achievement test scores were determined to have a significant relationship with van Hiele levels. Grade point average, grade level, and gender were determined to have no significance within any of the groupings. Appendices include Van Hiele Geometry Test, Entering Geometry Test, Exiting Geometry Test, and answer sheets. Contains 28 references. (Author/MKR)

ED 372 924

The Significance of External Variables as Predictors of Van Hiele Levels in Algebra and Geometry Students

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Abstract

External Variables as Predictors
of Van Hiele Levels in Algebra and Geometry Students

by Jeffrey Frykholm

The purpose of the present study was to determine the extent to which factors of age, gender, grade point average, standardized achievement test scores, and geometry achievement acted as predictors for van Hiele levels in two groups of subjects either entering geometry (algebra students), or exiting geometry. Van Hiele levels were determined for all subjects (N = 328), with scores ranging from no level (level 0), to level 5. Average van Hiele levels (VHL) for the groupings were as follows: all subjects, VHL = 2.0; algebra students, VHL = 1.55; geometry students, VHL = 2.46.

Multiple Regression Annalysis results revealed that there was a significant relationship between the group of independent variables and the the dependent variable (van Hiele level) for all subjects tested, as well as within both the algebra and geometry subroups.

Individual variables determined to be significant with van Hiele levels among all subjects tested were class level (geometry or algebra), standardized achievement test scores, and geometry achievement test scores. For the geometry subgroup, significant relationships included the variables of

age, standardized achievement test scores, and geometry achievement test scores. Interestingly, age was determined to have a negative correlation. For the algebra subgroup, only the geometry achievement tests scores were determined to have a significant relationship with van Hiele levels.

Grade point average, grade level, and gender were determined to have no significance upon van Hiele levels within any of the groupings.

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Introduction

In the attempt to improve the quality of geometry instruction in schools, researchers and teachers alike have given considerable attention to the van Hiele theory of geometry acquisition and development throughout the past two decades. The pioneering work of Pierre van Hiele and his wife Dina van Hiele-Geldof revealed a series of cognitive levels through which every geometry student passes. These levels, hierarchical in nature and characterized by different ways of perceiving and structuring information (van Hiele, 1959b), describe the variety of ways people think about geometry. According to the van Hiele's, the learner, assisted by appropriate instructional experiences, passes through up to five levels of development. These levels have been defined by van Hiele (1959) and formulated well by Hoffer (1981) and others.

Problem Statement

The intent of this particular project is to focus not only on the van Hiele levels of students in geometry and algebra classes, but how the identification of a student's level is necessary and vital in providing significant learning opportunities for learners in mathematics classrooms. Whether or not this identification can be made in advance of any formal interaction with geometry, based on readily available information, is explored.

The research study described below addresses the extent to which age, gender, grade point average, standardized achievement test scores, and geometry achievement act as predictors for van Hiele levels. It examines the relationship between these five factors

and the van Hiele level in students either entering or exiting a geometry course. Specifically, the purposes of the study include the following:

Purpose 1: The primary purpose for the study is to determine whether or not factors of age, grade level, gender, grade point average, standardized achievement test scores, geometry achievement and class (algebra or geometry) correlate significantly with students' van Hiele level of development in geometry.

Purpose 2: A second purpose for the study is to determine the approximate van Hiele level of the average incoming geometry student.

Purpose 3: A third purpose for the study is to determine the approximate van Hiele level of the average exiting geometry student.

Method

Subjects

The participants consisted of 398 male and female geometry and algebra students (mean age = 15.12) from three middle schools and two high schools in Spokane, Washington, an urban center with population close to 177,000. The students, eighth through eleventh in grade level, ranged from thirteen to eighteen years in age. The testing was conducted during the last four weeks (May-June) of the 1991-92 school year.

Subjects were selected based upon the math class in which they were enrolled. In the high schools, only those students enrolled in either first year algebra, or first year geometry were targeted for selection. The study sought to test students of all abilities, and because the algebra and geometry classes are mainstreamed, (i.e., the class contains students ranging from high to low ability), it was possible to select full algebra or geometry classes of students with the knowledge that a broad representation of students with differing abilities would be chosen.

The middle school selection process was slightly different in the sense that only those eighth grade students who have demonstrated consistent proficiency in mathematics are given the opportunity to enroll in algebra. Therefore, it is generally accepted that the eighth-grade students in algebra represent above average, middle school students. Out of the 398 students initially tested in the project, seventy-seven (19.3%) were eighth grade students.

Therefore, roughly one-fifth of the students were known to be of above average ability at the outset of the study.

Selection of subjects by class was also effective for organizational purposes. To facilitate the testing of a large number of students within a limited amount of time, it was necessary to use the normal classroom setting as a structure for testing.

Though 398 subjects were initially tested, 17.6% of the subjects failed to meet necessary standards for inclusion in the final analysis. A total of seventy subjects were eliminated throughout the various stages of the study, reducing the sample to 328 subjects. While 17.6% appears to be a large portion of the subjects, it is not surprising given the parameters of the study, and the large number of factors to be correlated for each subject (class, age, gender, grade level, geometry knowledge test, grade point average, achievement test score). Those students for whom one or more of the previously mentioned factors could not be determined were eliminated from the study as described below.

Subjects were removed from consideration in the final analysis in one of three areas. First, for 4.8% of the subjects (N = 19, eight female, eleven male), unusual inconsistency in answers on the van Hiele test resulted in a pattern for which no van Hiele level could be determined. (For. example, since the van Hiele levels are hierarchical in nature, it is inconceivable that a student would attain level three without first attaining level two.) As a result, they were eliminated

from the data analysis. Incidently, this percentage is considerably lower than the 12% found unclassifiable by Usiskin (1982).

Secondly, due to the fact that testing was done on two days, student absenteeism became a significant factor. Of the 398 students that took the initial van Hiele test, twenty-nine (7.3%) were absent for the following test, which eliminated them from the study.

Finally, twenty-two (5.5%) students were eliminated because either their grade point average or their most recent achievement test scores could not be located in existing school records.

The subjects were divided in two groups depending upon the math class in which they were currently enrolled (represented by "class" variable). All subjects in the study were either enrolled in first year algebra, or first year geometry. In Spokane School District 81, the sequence of math classes places algebra immediately before geometry. Hence, for those students exiting algebra, the next math class to be taken is geometry. Algebra students receive little or no formal instruction in geometry throughout the school year.

The first grouping (N = 164) consisted of those students presently completing a course in first year algebra. Although the original number of algebra students tested was 216, fifty-two students were eliminated for reasons stated above. This algebra group (ALG-1) consisted of eighth, ninth, tenth, and eleventh grade students from the three middle schools, as well as students from the two high schools.

The second grouping of students (N = 164) consisted of students completing a course in first year geometry. Although the original number of geometry students tested was 182, eighteen students were eliminated for reasons stated above. This geometry group (GEO-1) consisted only of ninth, tenth, and eleventh grade students from the two high schools.

A breakdown of students based upon subject group, school, gender, and grade level is illustrated in tables one and two below.

Materials

The materials for the study consisted of three geometry tests. The first test, on which the study was centered, was the Van Hiele Geometry Test (used with permission of the University of Chicago).

Table 1

ALG-1 Groupings by School, Gender (M=Male, F=Female), and Grade level

	8th grade		9th grade		10th grade		11th grade	
school	M	F	M	F	M	F	M	F
Middle School A	12	9						
Middle School B	11	20						
Middle School C	12	13						
High School 1			6	10	5	7		
High School 2			16	21	7	10	1	4

Table 2

GEO-1 Groupings by School, Gender, and Grade Level

	9th grade		10th grade		11th grade	
school	M	F	M	F	M	F
High School 1	12	18	33	25	10	7
High School 2	7	10	19	14	5	4

The Van Hiele Geometry Test

Understanding the order and characteristics of the van Hiele theory is much easier than actually assigning a student a level based on testing and evaluating. Several tests have been designed to measure the van Hiele level of a student. The most valid form of testing is one-on-one questioning and answering involving the researcher and the student. Dina van Hiele (1957) spent a great deal of time throughout the course of several years to test a large number of students in this fashion to support her earliest hypotheses about level development.

However, in a study such as the present one, one-on-one individual testing of such a large number of students is not feasible, given a limited amount of time to collect and analyze data. Similarly, the Cognitive Development and Achievement in Secondary School Geometry project (CDASSG; Usiskin, 1982) found it necessary to develop a new test which retained the integrity of the van Hieles'

original tests, yet could be administered and analyzed for a large number of students over a short perioc' of time. The project, in which over 2600 students were tested, used a 25 question, multiple-choice test. This particular test has been selected for use in the current study with the permission of Usiskin and the University of Chicago.

Usiskin's modification, the Van Hiele Geometry Test, consists of five subtests, each of which contains questions written to correspond directly to statements from the van Hieles about characteristics students exhibit at each level (Usiskin & Senk, 1990). It has been the most frequently used instrument in assessing geometry readiness, (Usiskin & Senk, 1990), and has enjoyed a popular reputation as a reliable measurement tool. Reliability measures for the test appear in table three below.

Table 3

Kuder-Richardson and Horst Reliability Figures for the Van Hiele Test

Van Hiele Level	Kuder-Richardson	
Level 1	.39	.43
Level 2	.55	.59
Level 3	.56	.59
Level 4	.30	.31
Level 5	.26	.27

Note. Source: Usiskin, 1982.

One reason for the low reliabilities is the small number of test items in each subtest. Similar tests with twenty-five test items at each level would have reliabilities (by level) .79, .88, .88, .69, and .65 (Usiskin, 1982). The particularly low reliabilities at levels four and five perhaps represent the lack of specification of the van Hiele theory at these levels (Usiskin, 1982).

Each subtest corresponds to one of the five levels, and a student is assigned a van Hiele level based on the sequence of subsets mastered. Mastery at a given level is determined by answering correctly either three or four of the five questions at that level. Of course, the researcher must make a decision about which criterion to use with the knowledge that the chances of making a Type I error are greater when the three of five criterion is used, and the chances of making a Type II error are greater when using the four of five criterion.

The thirty-five minute timed test, (described previously and located in the appendix), consists of twenty-five multiple-choice questions (each test question having five options to choose from) representing the five different levels of geometry understanding as identified by the van Hieles.

The second test was designed to test the general geometry knowledge of students completing algebra and preparing to take geometry. This "entering geometry" test (ENT) was also reproduced with permission of the University of Chicago for use in the current

study. It consisted of twenty multiple-choice questions covering general geometry concepts. It had a twenty minute time limit.

The final testing device was designed for those students who had completed a year long course in geometry. This "exiting geometry" test (EXIT), also a twenty-minute, twenty-problem, multiple-choice test was developed from resource books which accompany the current geometry text book selected by the district.

It is important to note that there are differences between the EXIT and ENT tests. The intent of the two tests was to evaluate the geometry knowledge of subjects with respect to their course background. Obviously, the tests reflected the fact that a geometry student would know more about geometry than would an algebra student.

Reliability measures for the Van Hiele Test and the Entering Geometry test are reported from Usiskin (1982). The 20-item ENT test has Kuder-Richardson formula 20 reliability of .77. Further, Horst's modification gives .79 reliability, which would correspond to an .89 reliability had the test contained forty items (Usiskin, 1982).

The Van Hiele test, for purposes of reliability, was considered as five, 5-item tests (Usiskin,1982). The Kuder-Richardson and Horst modification reliabilities were given previously in table 2.

Procedure

Upon selection, subjects were informed that they were to be tested for the following reasons: 1) to see whether or not their

understanding of geometry was consistent with their performance on a geometry test; 2) to see if there were any predictors such as grade point average, age, gender, etc., that could indicate how much they knew about geometry and how successful they would be in geometry; 3) to determine how successful the school system is in teaching students the principles of geometry; and 4) to help teachers improve geometry instruction in the classroom.

Because the combined length of the two tests exceeded the amount of time in a class period, it was necessary to test the subjects on two different days. In every case, there was less than one week between the administration of the two tests.

Data Collection

The grading of the tests was completed manually by the researcher. For the ENT and EXIT tests, a raw score was given based upon the number of correct responses out of twenty test items. There was no penalty for wrong answers.

Because the van Hiele test is designed to determine a student's level of thought, it is graded differently than the two previously mentioned tests. Essentially, within the twenty-five questions, there are five subtests, each graded individually. A subject was considered to have mastered a level if three or more of the five questions per subtest were answered correctly. Hence, it was possible for a student to miss several questions, yet still attain level four or five. Again, by choosing the three-of-five criterion (rather than a four-of-five criterion), the chances of making a type-one error increase.

Hence, the probability that a small percentage of students were classified at a level higher than they actually are exists.

The rest of the data was either supplied by the student (age, grade level, gender), or through school records (grade point average, achievement test score). The grade point average (GPA) score reflected each subject's current, cumulative grade point average (starting in seventh grade), based on a 4.0 scale.

The achievement score was based on each subject's most recent, national, standardized achievement test. The score recorded was the national percentile for overall math achievement. Although it varied slightly for some students who had recently enrolled in Spokane School District, most students were tested in either their eighth or tenth grade year. Therefore, the achievement scores represented overall math proficiency in terms of a national percentile for either the current school year (eighth and tenth grade subjects) or for the previous year (the ninth and eleventh grade subjects). The two national achievement tests cited by the project are the California Achievement Test (CAT), and the Metropolitan Achievement Test (MAT), both highly regarded achievement tests. In order to keep the results of the study as consistent as possible, those students who did not have achievement test scores from either of these two tests were not considered in the final analysis.

Kuder-Richardson reliability coefficients for both the CAT and the MAT are illustrated below in tables four and five. Only the

reliability figures for the tests reflecting grades eight through eleven are given.

Table 4

Range of Kuder-Richardson Formula 20 Reliability Coefficients for the Metropolitan Achievement Test

Level	Reading	Mathematics	Language
Intermediate	.76-.95	.80-.91	.82-.92
Advanced	.77-.90	.75-.91	.79-.91

Note. Source: Test Critiques, vol. III, 1985.

Table 5

Reliability Coefficient Scores for the California Achievement Test

Level	Reading	Mathematics	Language
Grade 9	.87	.78	.74
Grade 11	.91	.92	.87

Note. Source: Test Critiques, vol. III, 1985.

Results

Because of the seven independent variables and three subgroupings (all subjects = ALL, algebra subjects = ALG-1, geometry subjects = GEO-1), the representation of the data can be done in a number of ways. The first part of the results section illustrates the results descriptively, while the second part of the section details the relationships between the independent and dependent variables through a multiple regression analysis.

<u>Descriptive Statistics</u>

Of interest in the study was the performance all subjects, as well as the comparison of the two subgroups. Table 6 below indicates the mean, standard deviation, and range values for the variables (age, grade level, achievement test score (ACH), grade point average (GPA), and van Hiele level (VHL)), across the ALL grouping.

Likewise, tables 7 and 8 reveal the descriptive statistics for the two subgroups, ALG-1, GEO-1. Coincidentally, N = 164 for both samples.

Most significant to the study were the results of the Van Hiele Test. The purposes of the study were to determine whether or not any of the independent variables significantly influenced the van Hiele level (VHL) of the subjects, as well as to compare VHL results between the ALG-1 and GEO-1 groups.

Table 6

Descriptive Statistics: Age, Grade, VHL, and GPA for Subjects in the ALL Grouping

Variable	mean	Std Dev	Minimum	Maximum
Age	15.12	1.05	13	18
Grade	9.32	.94	8	11
ACH	70.98	21.0	10	99
GPA	3.23	.65	.90	4.00
VHL	2.00	1.17	0	5

Note. N = 328; ACH values reflect national percentiles.

Table 7

ALG-1 Descriptive Statistics: Age, Grade, VHL, ENT, GPA for all Algebra Subjects

Variable	mean	Std Dev	Minimum	Maximum
Age	14.59	.98	13	17
Grade	8.77	.85	8	11
GPA	3.26	.66	.90	4.00
ACH	71.51	21.82	17	99
ENT	11.03	3.77	2	18
VHL	1.55	1.02	0	4

Note. N = 164; ACH values reflect national percentiles.

Table 8

GEO-1 Descriptive Statistics: Age, Grade, VHL, EXIT, GPA for all Geometry Subjects

Variable	mean	Std Dev	Minimum	Maximum
Age	15.65	.83	14	18
Grade	9.87	.66	9	11
GPA	3.20	.65	1.27	4.00
ACH	70.45	20.20	10	99
EXIT	11.96	3.71	4	20
VHL	2.46	1.13	0	5

Note. N = 164; ACH values reflect national percentiles.

For these purposes, tables 9 and 10 below illustrate how students scored on the van Hiele test by grade level. That is, tables 9 and 10 indicate how the subjects of each grade were distributed with respect to their VHL. The results are given both in numbers and percentages.

As expected with the ALG-1 group, only one student reasoned above level three. The largest concentration of subjects (34.1%) registered at level two.

Interestingly, of the 34 subjects at level three, 27 of them (79%) were eighth graders. Further, while 35% of the eighth grade students scored at level three, 33% scored at level 2. Hence, 68% of the eighth grade students scored above the mean level (1.55) within the ALG-1 subgroup. Relative to the ALG-1 group, performance of the eighth grade students was very strong.

The younger subjects in the GEO-1 group (table 10), in this case ninth graders, also appeared to score consistently higher than their older counterparts. Of the subjects scoring at level five, 70% were ninth graders. Of the 21 subjects scoring at either level four and five, thirteen (62%) were ninth graders, the rest tenth graders. Interestingly, only one student GEO-1 subject, an eleventh grader, failed to reach level one.

As expected, comparisons of tables 9 and 10 indicate that the GEO-1 group demonstrated higher levels of performance on the van Hiele Test. Compared to 21.3% of the ALG-1 group, 52.4% of the

GEO-1 group reasoned at level three or higher. The majority of the subjects (39.6%) reasoned at level three.

Table 9

Algebra Group: Frequency/Percentage of Subjects at Van Hiele Levels by Grade Level

Grade	Level 0	Level 1	Level 2	Level 3	Level 4	Level 5	total & column %
8	4	20	26	27	-	-	77
	5.2%	26.0%	33.8%	35.0%			46.9%
9	15	19	13	5	1	-	53
	28.3%	35.8%	24.5%	9.4%	1.9%		32.3%
10	7	16	4	2	-	-	29
	24.1%	55.2%	13.8%	6.9%			17.7%
11	1	1	3	-	-	-	5
	20%	20%	60%				3%
Total	27	56	46	34	1	-	164
	16.5%	34.1%	28%	20.7%	.6%		100%

Note. With the exception of the final column, all percent figures represent row percentages.

Table 10

Geometry Group: Frequency/Percentage of Subjects at Van Hiele Levels by Grade Level

Grade	Level 0	Level 1	Level 2	Level 3	Level 4	Level 5	Total & Column %
9	-	7	5	22	6	7	47
		14.9%	10.6%	46.8%	12.8%	14.9%	28.7%
10	-	23	26	34	5	3	91
		25.3%	28.6%	37.3%	5.5%	3.3%	55.5%
11	1	9	7	9	-	-	26
	3.8%	34.6%	26.9%	34.6%			15.9%
Total	1	39	38	65	11	10	164
	.6%	23.8%	23.2%	39.6%	6.7%	6.1%	100%

Note. With the exception of the final column, all percent figures represent row percentages.

Tables 11 and 12 below illustrate subjects at each van Hiele level with respect to age. Table 11 distributes the algebra subjects across the van Hiele levels by age, and table 12 distributes the geometry subjects across the van Hiele levels by age. Once again, for both subgroups, the general trend was for the scores of the younger subjects to be higher than those of the older subjects.

Table 11

Algebra Group: Number/Percentage of Subjects at Van Hiele Levels by Age

Age	Level 0	Level 1	Level 2	Level 3	Level 4	Level 5	Total & Column %
13	1	3	8	5	-	-	17
	5.9%	17.6%	47.1%	29.4%			10.4%
14	5	20	18	23	-	-	66
	7.6%	30.3%	27.3%	34.8%			40.2%
15	15	21	13	3	1	-	53
	29.4%	41.2%	25.5%	5.9%	1.9%		32.3%
16	4	9	6	3	-	-	22
	18.2%	40.9%	27.3%	13.6%			13.4%
17	2	3	1	-	-	-	6
	33%	50%	16.7%				3.7%
total	27	56	46	34	1	-	164
	16.5%	34.1%	28%	20.7%	.6%		100%

Note. With the exception of the final column, all percent figures represent row percentages.

Table 12

Geometry Group: Number/Percentage of Subjects at Van Hiele Levels by Age

Age	Level 0	Level 1	Level 2	Level 3	Level 4	Level 5	Total & column %
14	-	-	-	7	2	2	11
				63.6%	18.2%	18.2%	6.7%
15	-	10	14	26	5	6	61
		16.4%	22.9%	42.6%	8.2%	9.8%	37.2%
16	-	17	20	25	4	2	68
		25%	29.4%	36.8%	5.9%	2.9%	41.5%
17	1	12	3	7	-	-	23
	4.3%	52.2%	13%	30.4%			14%
18	-	-	1	-	-	-	1
			100%				.6%
Total	1	39	38	65	11	10	164
	.6%	23.8%	23.2%	39.6%	6.7%	6.1%	100%

Note. With the exception of the final column, all percent figures represent row percentages.

Multiple Regression Analysis

Given the data as described above, additional statistical tests were necessary to determine what, if any, value the study had with respect to the original purposes. Given the constructs of this study, a multiple regression analysis was used.

A multiple regression analysis (MRA) is a statistical method useful in explaining the relationship between a dependent variable and two or more independent variables. The purposes for applying MRA tests are to evaluate hypotheses concerning the relationships between a dependent variable and a set of independent variables, to generate an equation useful in predicting the dependent variable from the independent variables, or to do some combination of these things. For this study, the relationship between a subject's van Hiele level as determined by the Van Hiele Test (dependent variable) and a series of independent variables will be examined. Among other things, the study intends to determine if there are any significant predictors of van Hiele levels.

Specifically, MRA is used to generate R^2, an index of the proportion of variation in a dependent variable that is predictable from a set of independent variables. Following the calculation of R^2, statistical tests of significance using the *F* distribution (Analysis of Variance (ANOVA)), as well as the standardized partial regression coefficient will be established.

Table 13 below lists the values of R, R^2, and Standard Error (SE) determined for each grouping, ALL, ALG-1, and GEO-1. The value of

R^2 described above originates in the calculation of the value of R. While the multiple correlation coefficient, R, estimates the magnitude of the relationship between the dependent variable and a linear combination of independent variables, R^2 provides a measure of the proportion of variation in the dependent variable accounted for by the set of independent variables. The SE measure represents the percent of scores falling within the first standard deviation (+/-) of the mean.

The value of R^2 can be roughly translated into words. Taking the R^2 value for the GEO-1 group, .32670, for example, the following relationship can be expressed: 32.67% of the variability in VHL is accounted for by the composite of the independent variables.

Table 13

Multiple R, R Square, and Standard Error (SE) for ALL, ALG-1, GEO-1 Groups

Group	R	R^2	SE
ALL	.63112	.39831	.91472
ALG-1	.52631	.27701	.88117
GEO-1	.57157	.32670	.94606

The amount of significance each individual variable has within this amount, as well as the issue of how influential chance differences are in the MRA will be discussed in the following pages.

An analysis of variance (ANOVA) using the *F* distribution test of significance was applied to the data to address the question of whether or not R^2 arose by chance, or whether it reflected a systematic relationship between the dependent variable, van Hiele level, and the independent variables. The *F* distribution is defined by the following equation:

$$F = \frac{(R^2/k)}{(1-R^2)/(N-k-1)}, \qquad (1)$$

where R^2 is the measure of the proportion of variation in VHL accounted for by the set of independent variables, 'k' is the number of independent variables, and 'N' is the sample size.

Equation two calculates the same *F* value using a slightly different formula. In equation two, the total variability in the dependent variable (R^2) has simply been split into two components. The first component (SS reg.), is predictable from the set of independent variables and called the regression sum of squares. The second component (SS res.) is unpredictable from the set of independent variables, called the residual sum of squares. Therefore, replacing the R^2 in equation one, equation two reads:

$$F = \frac{SS\ reg/k}{SS\ res/(N-k-1)}, \qquad (2)$$

where 'k' equals the number of independent variables, and 'N' equals the sample size.

Tables 14, 15, and 16 below illustrate the analysis of variance for the three groupings.

Table 14

ANOVA on VHL for ALL subgroup

	DF	Sum of Squares	Mean Squares	F	Signif.
Regression	7	177.24674	25.32096	30.26219	0.0
Residual	320	267.75021	.83672		

Table 15

ANOVA on VHL for ALG-1 Subgroup

	DF	Sum of Squares	Mean Squares	F	Signif.
Regression	6	46.70598	7.78433	10.02545	0.0
Residual	157	121.90378	.77646		

Table 16

ANOVA on VHL for GEO-1 Subgroup

	DF	Sum of Squares	Mean Squares	F	Signif.
Regression	6	68.18194	11.36366	12.69644	0.0
Residual	157	140.51928	.89503		

For all three of the above subgroups, a significant relationship exists. That is, the combination of independent variables significantly impacted the VHL within each subgroup. To be considered significant, the observed *F* value must be greater than the critical *F* value for identical sample size and degrees of freedom.

Incidently, determination of the degrees of freedom in the MRA is slightly different than in most statistical tests. In the case of the MRA, the degrees of freedom represent the total number of independent variables. Hence, the degrees of freedom for table 14 (ALL group) represent the seven independent variables to be analyzed with respect to the dependent variable, VHL. Specifically, those independent variables are: class (geometry or algebra), achievement test, gender, EXIT/ENT geometry test, age, grade, and GPA. Since the "class" variable was used to separate the subjects into two groups, it was removed from consideration in tables 15 and 16. Hence, the degrees of freedom for tables 15 and 16 is one less (6) than table 14.

One purpose of MRA is to provide an equation useful in predicting the value of the dependent variable from given independent variables. This predicted value of the independent variable (Y) is equal to a linear combination of the dependent variable X's. For each of the X values (in this case age, GPA, gender, etc.), there corresponds a particular weight, '*b*".

These weights are determined so as to provide the best mathematical prediction of Y. Called partial regression coefficients, the *b* 's, show the relationship between the dependent variable (Y), and the corresponding independent variable (X).

The general multiple regression equation for predicting a dependent variable from *k* independent variables appears below in equation three:

$$Y = a + b_1X_1 + b_2X_2 + \ldots + b_kX_k \tag{3}$$

As in all multiple regression equations, constant value 'a' is estimated by least square calculations.

In the study, three multiple regression equations were formulated, one for each of the subgroupings. In each equation, the dependent variable, Y, refers to the van Hiele level. All other variables represent independent variables. Table 17 below indicates the '*b*' value for each of the independent variables by grouping. The 'a' value for each equation appears as 'constant' in the table.

Thus far, MRA results indicate significant relationships between the VHL and the independent variables for all three

groupings of subjects. Further, a multiple regression equation for the three groups has been established. Unanswered thus far is the question of which, if any, individual variables can be considered significant indicators of a subject's van Hiele level. The answer to this question is discussed below.

A final statistical test was applied to the data to test the variance of competing independent variables. That is, the *F* statistic was again used to determine which, if any, of the independent variables significantly increase the predictability of VHL. Table 18 includes the *F* value for each independent variable.

Further, table 18 also includes the standardized partial regression coefficients (Beta) for each independent variable. Beta accounts for large differences in the variances of individual independent variables. This value, Beta, standardizes the variances of the variables to 1.00. Hence, the larger the absolute value of Beta for an independent variable, the stronger predictor the variable is.

Highlighting only those variables with significant relationships, table 19 indicates, by grouping, the variables having significant predictive value on VHL. The *F* value for each variable, as well as accompanying probability values are included. Critical values ($p < .05$) of *F* were determined to be F (critical) = $F(.05, 1, 157) = 3.91$ for the ALG-1 and GEO-1 groups, and F (critical) = $F(.05, 1, 320) = 3.87$ for the ALL group. Hence, only those variables whose *F* values were greater than these figures were considered to be significant.

In a final display of data, the matrix in table 20 illustrates the simple correlation coefficient for all pairs of variables. Generated as part of the MRA, the correlation coefficients come from the Pearson-r product moment correlation model. Simply put, the values within the matrix indicate the degree of relationship between the two variables on a scale of zero to (plus or minus) one.

Table 17

Coefficients of the Independent Variables in the Multiple Regression Equations (VHL = Dep. Variable) for Groups ALL, ALG-1, and GEO-1

Independent Variables	Groupings		
	ALL	ALG-1	GEO-1
Constant	-.069918	-.453920	3.534192
Class	.901536	* *	* *
Ach. Test	.011553	.010030	.013494
Gender	.118017	.209531	.029672
EXIT/ENT	.095475	.093288	.098699
Age	-.168801	.001882	-.314827
Grade	.110108	-.020680	.153819
GPA	.046505	.026329	.048798

Note. Subgroups ALG-1 and GEO-1 were separated by the level variable; i.e., the "class" index separated Algebra and Geometry subjects. Hence, it was not considered an independent variable in the multiple regression equation for the ALG-1 and GEO-1 subgroups.

Table 18

ANOVA for Independent Variables by Groupings, Citing the Standardized Partial Regression Coefficient (Beta) and F-value

Ind Var	Groupings					
	ALL		ALG-1		GEO-1	
	Beta	F	Beta	F	Beta	F
Class	.387	**38.523**	-	-		
ACH	.20797	**11.264**	.21518	3.443	.240861	**8.604**
Gender	.05065	1.211	.102831	1.959	.013136	.035
EXIT/ENT	.3079	**35.447**	.345657	**16.602**	.323547	**18.346**
GPA	.02026	.208	.017018	.031	.027973	.117
Age	-.15174	2.714	.001809	.000	-.230103	**4.327**
Grade	.08849	.644	-.017241	.012	.089281	.603

Note. **Boldface** indicates significance at the $p < .05$ level;

The class variable was used to separate the two subgroups. Hence, for the ALG-1 and GEO-1 groups, it was not considered in the analysis.

Table 19

F and Probability Values for Significant Independent Variables by Groupings

Ind Var	Grouping					
	ALL		ALG-1		GEO-1	
	F	Prob/Sig	F	Prob/Sig	F	Prob/Sig
Class	38.523	.000	-	-	-	-
ACH	11.264	.0009	-	-	8.604	.0039
EXIT/ENT	35.447	.000	16.602	.0001	18.346	.000
AGE	-	-	-	-	4.327	.0391

Table 20

Pearson-r Correlation Coefficients for All Variables

	Class	Grade	Age	Gendr	ENT/EX	VHL	GPA	ACH
Class	1.00	-	-	-	-	-	-	-
Grade	.5898	1.00	-	-	-	-	-	-
Age	.5066	.8798	1.00	-	-	-	-	-
Gendr	-.7032	-.1159	-.0959	1.00	-	-	-	-
ENT/EX	.1234	-.2409	-.2657	-.0247	1.00	-	-	-
VHL	.3900	-.0233	-.0853	.0078	.4812	1.00	-	-
GPA	-.0499	-.4945	-.4556	.2594	.3707	.2600	1.00	-
ACH	-.0253	-.5456	-.5238	-.0125	.4840	.3904	.4387	1.00

Discussion

From the outset, this study was designed with high school geometry teachers in mind. It was intended to provide teachers with some meaningful information regarding the level of ability of students likely to be enrolled in geometry classes. Further, it was hoped that the study might provide some ideas as to how mathematics curriculum could be developed to facilitate student success in geometry. With these general considerations in mind, the following pages are intended to discuss the results of the study with regard to the project hypotheses as well as the interests of the geometry classroom as stated above.

Group Differences

One of the primary purposes of the study was to determine the level of ability for the average incoming geometry student. Of course, there is no one index that can by itself fully determine such a measure. Many factors contribute to the success or failure of individual students. Bearing this caution in mind, however, it is fair to say that, among other considerations, the van Hiele theory can be of significance in determining how ready a student is to be exposed to different aspects of geometry. As detailed earlier, the van Hiele levels indicate how familiar students are with basic geometry concepts, as well as how capable they are of functioning within a deductive system of thought.

Algebra Group

The purpose of the ALG-1 group was to test incoming geometry students, as well as to offer a comparison to those students completing a year long course in geometry. Concurring

with Wirszup (1976), Burger & Shaughnessy (1986), and Usiskin (1982), the majority of entering geometry subjects (ALG-1) were at or below the second van Hiele level. Specifically, the average entering van Hiele level was 1.55, with 74% of the students at level two or less. Moreover, fully half of the AGL-1 group, 50.6%, were at level one or less.

These numbers, at their simplest interpretation, provide evidence that a large majority of entering geometry students have very little background in the field of geometry. Again, this statement must be made with caution. Because the students averaged a van Hiele level of 1.55 does not mean they are incapable of learning geometry. More likely, it means that they have had little exposure to it.

This information should serve as a warning to all geometry teachers. To meet incoming students at the appropriate level of difficulty may mean beginning the year more slowly and with basic introductory material. Unfortunately most geometry classes are not taught at lower levels. Rather, as indicated by Wirzup (1976), most geometry teachers begin instructing at level four.

Geometry Group

If students enter at or below level two, the natural question that follows is, "At what level do they exit?". This issue was another one of the primary objectives of the study. If van Hiele (1959a) is correct in asserting that instruction is the most important factor in improving the level of a learner, then one would expect a significant increase in the van Hiele levels of the geometry students over those of the algebra students.

The results of the study support the fact that the geometry students were able to reason at higher levels. The average van Hiele level for the 164 geometry students was 2.46. Initially, it appears that the difference in average scores between the two groups is not as much as perhaps expected (2.46 - 1.55 = .91). A *t-test* for independent samples, however, indicates a significant gain in VHL from the AGL-1 group to the GEO-1 group [$t(326) = 3.54$; $p < .05$].

From a slightly different perspective, further evidence of the disparity between groups appears. While 50.6% of the ALG-1 group reasoned <u>at or below</u> level one, 52.4 of the GEO-1 group reasoned <u>at or above</u> level three. That is, over half of the incoming geometry students could not reason above the first level, while over half of the exiting geometry students reasoned at or beyond the third level.

<u>Age Differences</u>

The fact that the geometry students were at higher levels is no surprise. Although the expected difference was documented, there still persist questions as to how much difference should have been expected. Is a difference in averages of .91 near the amount one should expect in this case? If so, was it the instruction that caused this increase, or other factors? How did the fact that most classes are taught at levels beyond which the students are capable of reasoning impact the scores? One wonders what the gain would have been had the participating teachers known of both the theory and van Hiele levels of their students throughout the year. Perhaps it would have been much greater; perhaps it would have remained the same.

Because the current project was not designed to answer these questions, many of them remain unanswered. Some of the data,

however, does provide some interesting insights related to these questions. One particular area of interest is evident when looking at the van Hiele scores as distributed by age. Perhaps defying what seems logical, younger subjects appeared to score better than their older classmates.

For example, within the ALG-1 group, the 164 subjects were divided almost evenly between 13 and 14 year olds, and 15, 16, and 17 year olds. There were 83 subjects either 13 or 14 years old, and 81 subjects either 15, 16, or 17 years old. Though these two subgroups are nearly identical in size, the way their scores were distributed is surprising.

Specifically, of the 35 ALG-1 subjects to score at level three or four, 28 of them (80%) were 13 or 14 years old. Only 20% of the top students in the ALG-1 group were 15, 16, or 17 years old. At the other end of the spectrum, levels zero and one, a similar trend is observed, only reversed. Here, the majority of the lowest students were the older ones. Of the 83 subjects that scored at level one or below, 54 of them (65%) were the older 15, 16, and 17 year-old subjects. Thirty-five percent of the lowest scores were for 13 and 14 year-olds. One final contrast indicates that, while 23.5% of the 13 year-olds were at or below level 1, 59.1% of the 16 year-olds and 88% of the 17 year-olds scored at the lowest levels. Clearly, within the ALG-1 group, younger students outscored older ones.

This trend was also observed in the GEO-1 grouping. Of the 164 subjects, 72 were either 14 or 15 years old. These subjects made up the majority of the scores at the top two levels. Of the ten subjects at level 5, eight of them (80%) were either 14 or 15 years

old. At level 4, seven of the eleven subjects were again 14 or 15 years old. Further, no 17 or 18 year-olds scored at either level 4 or 5. In looking at the 14 year-olds alone, 36.4% were at level 4 or 5 compared to only 8.8% of the 16 year-olds.

Conversely, no 14 year old scored <u>below</u> level 3, while 54.4% of the 16 year-olds, 69.5% of the 17 year-olds, and 100% of the 18 year- olds scored below level 3.

These figures taken and adjusted from tables 11 and 12 suggest there might be some relationship between age and van Hiele level. This relationship was in fact discovered to be significant through the multiple regression analysis discussed below. Even had the relationship not been determined to be significant, however, there still would be grounds to suspect the younger students would score better.

Already hinting at this relationship, table 18 indicated that there was a negative correlation between age and VHL. By looking closely at the table, the Beta values for the age variable under the ALL and GEO-1 groupings are negative. This negative sign means that the relationship, although not yet necessarily significant, is negative. That is, in contrast to a positive relationship in which an increase in the independent variable means an increase in the dependent variable, a negative relationship indicates that a <u>decline</u> in the independent variable actually <u>increases</u> the dependent variable.

The rationale for this relationship comes from the fact that in the district where the testing occurred, geometry is open for enrollment to a wide range of students. Some of the students enrolled in geometry are taking the class as a final graduation

requirement in mathematics. Hence, the older students tend to be those who have been out of math for one or two years, and are simply taking the geometry class for the final math credit.

Conversely, those students taking geometry as freshmen are considered to be the top math students coming out of the middle schools. In order to be in the accelerated track in mathematics (culminating in calculus at the senior level), freshman geometry is required. Hence, the freshman students in geometry tend to be well trained and confident in their mathematical abilities.

Prediction

As stated at the outset, the main intent of the project was to determine whether or not there were any factors which could significantly predict the van Hiele level of a student. The results of the multiple regression analysis as reported earlier did indeed reveal several significant relationships.

Before discussing the individual variables that correlate significantly with the van Hiele levels, it is necessary to look at the effect of the entire group of independent variables upon the dependent variable, VHL. With any multiple regression analysis, one must always ask the question of whether variations in the dependent variable are due to the affect of the independent variables, or whether they are due to chance differences.

As tables 14, 15, and 16 indicate, *F* distribution results conclude that the observed relation between VHL and the other independent variables for the ALL, ALG-1, and GEO-1 populations was not due to chance; rather, it was due to a systematic relationship between van Hiele level and the group of independent

variables. In each subgroup, the observed value of *F* was greater than the corresponding critical value.

One way of determining just how much influence the group of variables had on VHL is to cite the values of R^2 for each analysis. Again, R^2 measures the proportion of variation in VHL accounted for by the independent variables. Therefore, when observing the R^2 values in table 14, the following statements may be made. First, for the ALL group, 39.8% of the variability in VHL in this group was dependent upon the influence of the group of independent variables. Secondly, for the ALG-1 group, 27.7% of the variability in VHL was dependent upon the independent variables. Finally, 32.67% of the variability for VHL in the GEO-1 group was dependent upon the independent variables.

The fact that the significance in the ALL group was greater than either of the subgroups at first seems puzzling. One would think that the proportion of variance in the ALL group could not be greater than both of the subgroups. This condition can be explained in that the ALL group contained one additional variable that the other two groups did not. Further, this particular variable, LEV, had the highest *F* value of all the independent variables in the study, indicating it had the highest significance upon VHL.

Intuitively, this relationship makes sense. The two subgroups (ALG-1 and GEO-1) were divided by "Class", and consequently the "Class" variable was not used in the analysis of these subgroups. That is the "Class" variable was used to sort students into two groups, one group consisting of the algebra students, and one group consisting of the geometry students. In comparing these two

groups, there was an obvious difference in the average van Hiele score. Hence, the "Class" variable, (whether or not a subject was in geometry or algebra), was very important in determining the VHL of the subject. In the MRA for the ALL group, then, "Class" became extremely significant, adding to the general predictability of the relationship.

The mechanics of this predictive process (equation) can be seen, again intuitively, by a simple example. Suppose that two pieces of information were to be revealed about a subject, and from them a prediction of the subjects van Hiele level was to be made. The MRA helps define which two pieces of information would be most helpful to know. Using the results of this study as examples, it would be of little value to know the gender and grade level of a student if we were to predict that student's van Hiele level. Boys and girls of various grades varied in their performance.

Rather, it would be most helpful to know at what level the subject was participating. That is, is he or she in algebra, or in geometry? Naturally, predicting the VHL would be much easier if it was known whether the student was receiving instruction in geometry, or if the student had received no instruction in geometry. Secondly, in light of the fact younger students appear to outscore older students, it would be very helpful to know the age of the student. For example, by knowing that a subject was in geometry and was also 14 years of age, it would be reasonable to predict that the student's van Hiele level was at least level three. Of course, this is only an example of how the prediction process works. The point

of the MRA, however, is to allow us to make the best prediction possible.

Significant Independent Variables

As tables 19 and 20 indicate, four of the seven variables are significant at one point or another. As detailed above, the LEV variable is significant within the ALL group, but does not apply to the other groups.

Only one variable was significant within all three groups. This independent variable was the score of the entering or exiting geometry test (ENT/EXIT). For the ALG-1 group, the entering geometry test was the only significant item useful in predicting VHL. It's F value was 16.602, well above the necessary critical value. In words, the only significant indicator of a subject's van Hiele level if he or she were known to be in algebra was the entering geometry test score. Again, this makes intuitive sense. If a student does well on a geometry exam, the chances are good that he or she has attained some of the van Hiele levels.

The exiting geometry test was significant in the GEO-1 group. Its F value was slightly higher than the ALG-1 score, at 18.346. It follows that the test score was also significant for the ALL group.

A second variable was shown to be significant among the ALL and GEO-1 groups. The standardized achievement score (ACH) had considerably less F values than the ENT/EXIT, yet was significant nonetheless. For the ALL group, $F = 11.264$. For the GEO-1 group, $F = 8.604$. It would be speculation to give any definite reason why the ACH variable was not significant in the ALG-1 group. As eluded to earlier, however, perhaps the majority of geometry students possess

greater general ability and academic seriousness than their algebra counterparts. It can be stated that not all students take the achievement tests seriously, and therefore the scores do not always reflect the capabilities of the subjects. Again, both of these assertions are only speculations.

The final significant variable, age level, has been discussed in the previous pages. It appears that age does have a significant relationship with VHL, but only in the GEO-1 group. The weakest of the significant variables, the F value for age was determined to be $F = 4.327$. Although it was the weakest compared to the other significant variables, it is still well above the critical value for F at the .05 level, and even above the value for F at the .01 level of significance.

Conclusion

Much of this study concurs with previous research on the van Hiele theory. As established earlier by others, this study confirms the fact that entering geometry students tend to be at or slightly below the second van Hiele level. Further, it is expected that through the course of a year's study of geometry, gains in the van Hiele level will occur. The average exiting geometry student tends to be slightly below the third level.

Notable in this study was the trend for younger students to outscore older students of the same level (algebra or geometry). While significant only in the GEO-1 group, Beta values in the ALL and GEO-1 sub-groupings indicate a negative relationship between age and VHL.

Finally, contributing to the research done thus far on the van Hiele theory, evidence to suggest that there are some valid predictors of VHL was revealed. While gender, grade level, and GPA appeared to have no significant relationship with van Hiele levels, entering and/or exiting geometry test scores, standardized achievement test scores, and age were all determined to have significant relationships with van Hiele levels.

These findings are important in a practical sense. The fact that age, geometry knowledge, and general math achievement are significant predictors of geometry ability is helpful in determining the likelihood of success for given students. Equally important, however, is the knowledge that some factors, specifically gender, grade level, and GPA, have little or nothing to do with a student's potential in geometry. For example, the knowledge that gender differences are not significant predictors of geometry achievement dispells the common myth that female students are not as capable of achieving high standards of excellence in math as their male peers. The fact that this has been documented statistically is meaningful information that should be discussed with both teachers and students.

Despite these findings, there still remains room for further research in this area. The fact that a large percentage of the variance in VHL remains unexplained (table 13) suggests that there could be other factors beyond the realm of the study that affect VHL. Identifying these factors, if they exist, could be helpful in explaining how students become successful learners in geometry. Further, continued experimentation with geometry curriculum at the

elementary level must be encouraged. Accelerating young learners through the beginning van Hiele levels can only help in the mastery of higher level geometry principles at the high school level. Finally, more research in the area of computer assisted instruction in geometry is necessary and valuable.

In closing, it was hoped by the researcher that this study would illuminate a mothod of predicting the inherent success of geometry students based upon the prediction of the van Hiele levels. It was hoped that if a method for predicting van Hiele levels could be established, the placement of students at appropriate levels of difficulty, as well as guidelines to curriculum and instruction, could be established.

Of course, one does not always get everything he or she hopes for. Although such a system for predicting the success of geometry students would be of great value, it can not yet be established. The results of the study, however, are enlightening in the same sense that the studies before it were. What is evident here, and should be revealed to all mathematics teachers, is that a wide range of students with various needs and abilities are present in every classroom. It should be of high priority to identify the level of each student as quickly as possible in order to meet that learner at the appropriate place.

Secondly, the implications for instruction remain the same. Mathematics educators must continue to provide stimulating classroom opportunities for learners to come in contact with the basic principles of geometry at young ages. Knowledge of the van Hiele levels, as well as knowledge of the steps through which

ascension of higher levels occurs should be a part of every math teacher's training.

Finally, students need to become aware of the process through which they learn mathematics, specifically geometry. By sharing with students how they become better thinkers, both the teacher and the student benefit. The student becomes aware of significant aspects of their own learning, and hence is able to develop and work in those areas. The teacher, through the sharing of such ideas, allows the student to burden some of the responsibility of their own learning. Perhaps most important, however, is the fostering of the student-teacher relationship. As van Hiele (1959) indicates, the relationship with the teacher is the most important element in a child's education. Any step a teacher can do to strengthen this relationship is, of course, a very wise and rewarding practice.

Appendix A

Van Hiele Geometry Test

TEST NUMBER ___

VAN HIELE GEOMETRY TEST

DIRECTIONS

Read each question carefully. Decide upon the answer you think is correct, and place that answer in the appropriate place on your answer sheet. There is only one correct answer to each question.

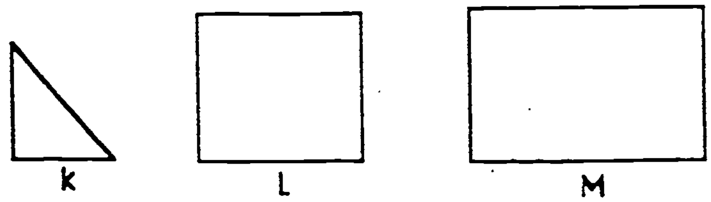
Use the blank sheets at the back of the test for scratch paper. Please do not write on this test.

You will have 35 minutes to complete this test. It is not expected that you will know the correct answer for all 25 questions.

VAN HIELE GEOMETRY TEST

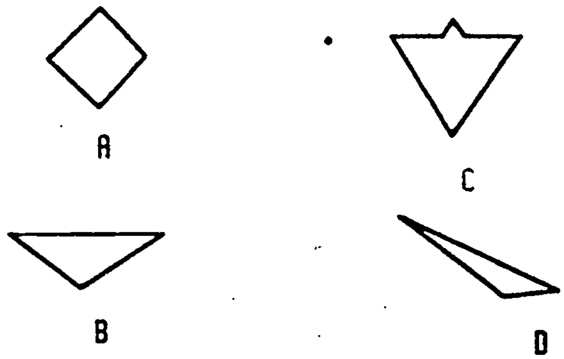
1. Which of these are squares?

(A) K only
(B) L only
(C) M only
(D) L and M only
(E) All are squares



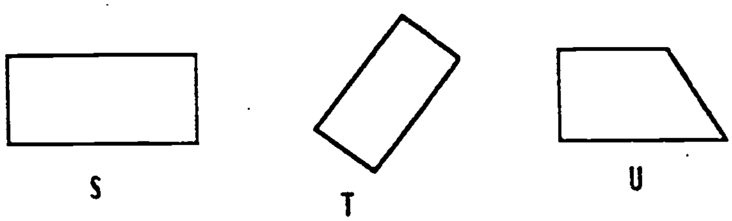
2. Which of these are triangles?

(A) none are triangles
(B) C only
(C) B only
(D) B and D only
(E) B and C only



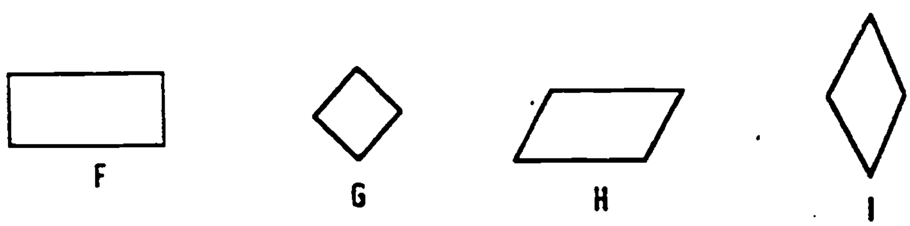
3. Which are rectangles?

(A) S only
(B) T only
(C) S and T only
(D) S and U only
(E) All are rectangles



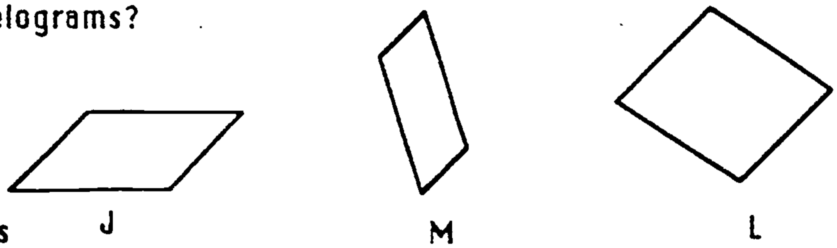
4. Which of these are squares?

(A) none are squares
(B) G only
(C) F and G only
(D) G and I only
(E) All are squares



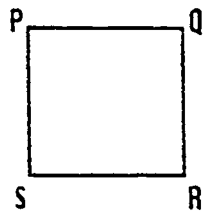
5. Which of these are parallelograms?

(A) J only
(B) L only
(C) J and M only
(D) none are parallelograms
(E) all are parallelograms



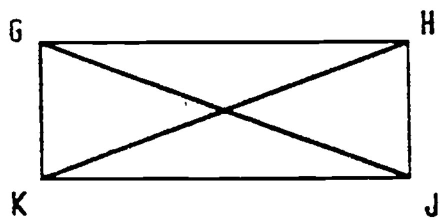
6. PQRS is a square.

Which relationship is true in all squares?



(A) PR and RS have the same length
(B) QS and PR are perpendicular
(C) PS and QR are perpendicular
(D) PS and QS have the same length
(E) Angle Q is larger than angle R.

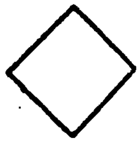
7. In a rectangle GHJK, $\overline{GJ}$ and $\overline{HK}$ are the diagonals.

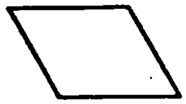


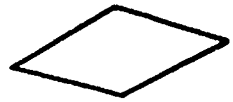
Which of the following (A - D) is not true in every rectangle?

(A) There are four right angles
(B) There are four sides
(C) The diagonals have the same length
(D) The opposite sides have the same length
(E) All of the above are true for every rectangle

8. A rhombus is a 4-sided figure with all four sides the same length. Here are three examples:


ex. 1

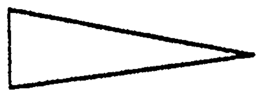

ex. 2


ex. 3

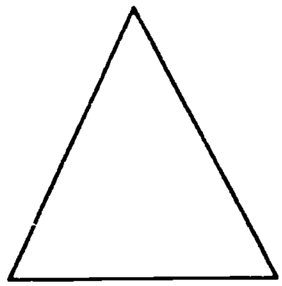
Which of the following (A - D) is not true for every rhombus?

(A) the two diagonals have the same length
(B) each diagonal bisects two angles of the rhombus
(C) the two diagonals are perpendicular
(D) the opposite angles have the same measure
(E) All of the above are true in every rhombus

9. An isosceles triangle is a triangle with two sides of equal length. Here are three examples:


ex. 1

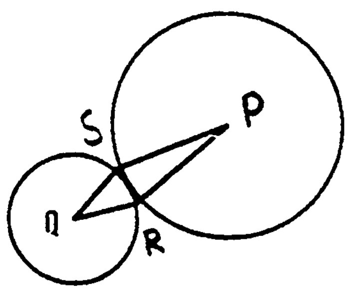

ex. 2


ex. 3

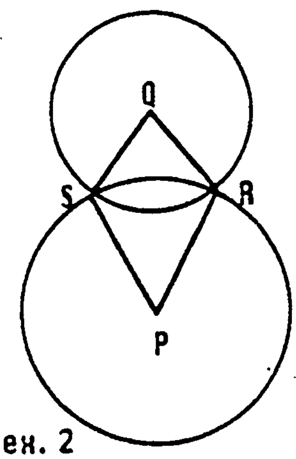
Which of the following (A - D) is true in every isosceles triangle?

(A) the three sides must have the same length
(B) one side must have twice the length of another side
(C) There must be at least two angles with the same measure
(D) three angles must have the same measure
(E) none of the above is true in every isosceles triangle

10. Two circles with centers P and Q intersect at R and S to form a 4-sided figure PQRS. Here are two examples:



ex. 1



ex. 2

Which of the following (A - D) is not always true?

(A) PQRS will have two pairs of sides of equal length
(B) PQRS will have at least two angles of equal measure
(C) the lines $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ will be perpendicular
(D) Angles P and Q will have the same measure
(E) All of the above are true

11. Here are two statements.

Statement 1: Figure F is a rectangle.
Statement 2: Figure F is a triangle.

Which is correct?

(A) If 1 is true, then 2 is true.
(B) If 1 is false, then 2 is true.
(C) 1 and 2 cannot both be true.
(D) 1 and 2 cannot both be false.
(E) None of (A) - (D) is correct.

12. Here are two statements.

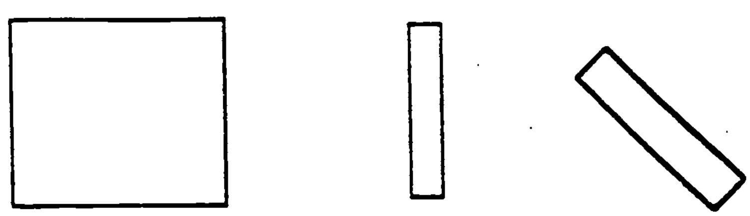
Statement S: Δ ABC has three sides of the same length.
Statement T: In Δ ABC, $\angle B$ and $\angle C$ have the same measure.

Which is correct?

(A) Statements S and T cannot both be true.
(B) If S is true, then T is true.
(C) If T is true, then S is true.
(D) If S is false, then T is false.
(E) None of (A) - (D) is correct.

13. Which of these can be called a rectangles?

(A) All can.
(B) Q only
(C) R only
(D) P and Q only
(E) Q and R only



P Q R

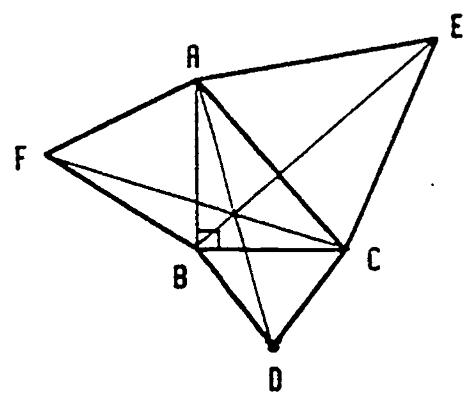
14. Which is true?

(A) All properties of rectangles are properties of all squares.
(B) All propeties of squares are properties of all rectangles.
(C) All properties of rectangles are properties of all parallelograms.
(D) All properties of squares are properties of all parallelograms.
(E) None of (A) - (D) is true.

15. What do all rectangles have that some parallelograms do not have?

(A) opposite sides equal
(B) diagonals equal
(C) opposite sides parallel
(D) opposite angles equal
(E) none of (A) - (D)

16. Here is a right triangle ABC. Equilateral triangles ACE, ABF, and BCD have been constructed on the sides of ABC.



From this information, one can prove that $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common. What would this proof tell you?

(A) Only in this triangle drawn can we be sure that $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common
(B) In some but not all right triangles, $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common
(C) In any right triangle, $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common
(D) In any triangle, $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common
(E) In any equilateral triangle, $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ have a point in common

17. Here are three properties of a figure.

Property D: It has diagonals of equal length.
Property S: It is a square.
Property R: It is a rectangle.

Which is true?
(A) D implies S which implies R.
(B) D implies R which implies S.
(C) S implies R which implies D.
(D) R implies D which implies S.
(E) R implies S which implies D.

18. Here are two statements.

1. If a figure is a rectangle, then its diagonals bisect each other.
2. If the diagonals of a figure bisect each other, the figure is a rectangle.

Which is correct?
(A) To prove 1 is true, it is enough to prove that 2 is true.
(B) To prove 2 is true, it is enough to prove that 1 is true.
(C) To prove 2 is true, it is enough to find one rectangle whose diagonals bisect each other.
(D) To prove 2 is false, it is enough to find one non-rectangle whose diagonals bisect each other.
(E) None of (A) - (D) is correct.

19. In geometry:

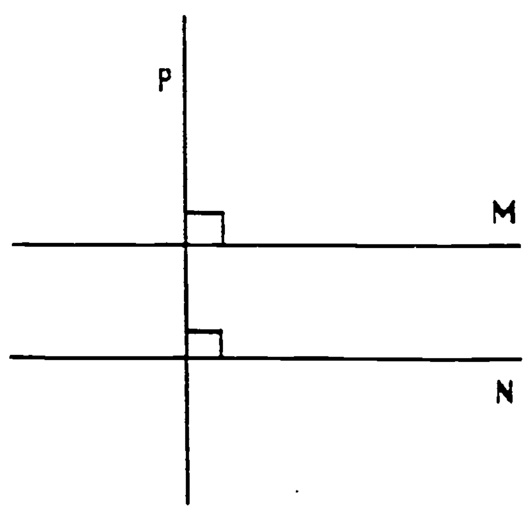
(A) Every term can be defined and every true statement can be proved true.
(B) Every term can be defined but it is necessary to assume that certain statements are true.
(C) Some terms must be left undefined but every true statement can be proved true.
(D) Some terms must be left undefined and it is necessary to have some statements which are assumed true.
(E) None of the above (A) - (D) is correct

20. Examine these three sentences.

(1) Two lines perpendicular to the same line are parallel.
(2) A line that is perpendicular to one of two parallel lines is perpendicular to the other.
(3) If two lines are equidistant, then they are parallel.

In the figure below, it is given that lines m and p are perpendicular and lines n and p are perpendicular. Which of the above sentences could be the reason that line m is parallel to line n?

(A) (1) only
(B) (2) only
(C) (3) only
(D) Either (1) or (2)
(E) Either (2) or (3)



21. In F geometry, one that is different from the one you are used to, there are exactly four points and six lines. Every line contains exactly two points. If the points are P, Q, R, and S, the lines are (P,Q) , (P,R) , (P,S) ; (Q,R) , (Q,S) , and (R,S)

Here are how the words "intersect" and "parallel" are used in F-geometry. The lines (P,Q) and (P,R) intersect at P because (P,Q) and (P,R) have P in common.

The lines (P,Q) and (R,S) are parallel because they have no points in common.

From this information, which is correct?
(A) (P,R) and (Q,S) intersect.
(B) (P,R) and (Q,S) are parallel.
(C) (Q,R) and (R,S) are parallel.
(D) (P,S) and (Q,R) intersect.
(E) None of (A) - (D) is correct.

22. To trisect and angle means to divide it into three parts of equal measure. In 1847, P.I. Wantzel proved that, in general, it is impossible to trisect angles using only a compass and an unmarked ruler. From his proof, what can you conclude?

(A) In general, it is impossible to bisect angles using only a compass and an unmarked ruler.
(B) In general, it is impossible to trisect angles using only a compass and a marked ruler.
(C) In general, it is impossible to trisect angles using any drawing instruments.
(D) It is still possible that in the future someone may find a general way to trisect angles using only a compass and an unmarked ruler.
(E) No one will ever be able to find a general method for trisecting angles using only a compass and an unmarked ruler.

23. There is a geometry invented by a mathematician J in which the following is true:

    The sum of the measures of the angles of a triangle is less than 180.

    Which is correct?

    (A) J made a mistake in measuring the angles of the triangle.
    (B) J made a mistake in logical reasoning.
    (C) J has a wrong idea of what is meant by "true".
    (D) J started with different assumptions than those in the usual geometry.
    (E) None of (A) - (D) is correct.

24. Two geometry books define the word rectangle in different ways. Which is true?

    (A) One of the books has an error.
    (B) One of the definitions is wrong. There cannot be two different definitions for rectangle.
    (C) The rectangles in one of the books must have different properties as those in the other book.
    (D) The rectangles in one of the books must have the same properties as those in the other book.
    (E) The properties of rectangles in the two books might be different.

25. Suppose you have proved statements 1 and 2.

    1. If p, then q.
    2. If s, then not q.

    Which statement follows from statements 1 and 2?

    (A) If p, then s.
    (B) If not p, then not q.
    (C) If p or q, then s.
    (D) If s, then not p.
    (E) If not s, then p.

Appendix B

Entering Geometry Test

Entering Geometry Test

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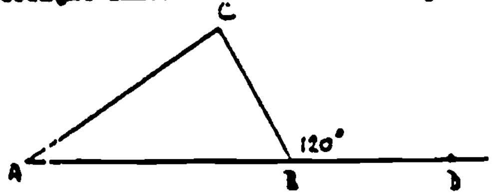
1. Perpendicular lines
   (a) intersect to form four right angles
   (b) intersect to form two acute and two obtuse angles
   (c) do not intersect at all
   (d) intersect to form four acute angles
   (e) none of the above

2. The area of a rectangle with length 3 inches and width 12 inches is
   (a) 18 sq in
   (b) 72 sq in
   (c) 36 sq in
   (d) 15 sq in
   (e) 30 sq in

3. If two figures are similar but not congruent then they
   (a) have congruent bases and congruent altitudes
   (b) have the same height
   (c) both have horizontal bases
   (d) have a different shape but the same size
   (e) have a different size but the same shape

4. The measure of an <u>obtuse</u> <u>angle</u> is
   (a) 90°
   (b) between 45° and 90°
   (c) less than 90°
   (d) between 90° and 180°
   (e) more than 180°

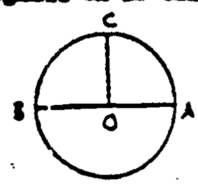
5. At right, A, B, and D lie on a straight line. The measure of angle ABC is
   (a) 120°
   (b) 60°
   (c) 80°
   (d) 240°
   (e) need more information



6. Parallel lines are lines
   (a) in the same plane which never meet
   (b) which never lie in the same plane and never meet
   (c) which always form angles of 90° when they meet
   (d) which have the same length
   (e) none of the above

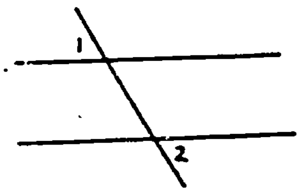
7. If O is the center of the circle, segment $\overline{OA}$ is called a

(a) radius of the circle
(b) diameter of the circle
(c) chord of the circle
(d) segment of the circle
(e) sector of the circle



8. Angles 1 and 2 are called

(a) opposite angles
(b) parallel angles
(c) alternate interior angles
(d) alternate exterior angles
(e) corresponding angles

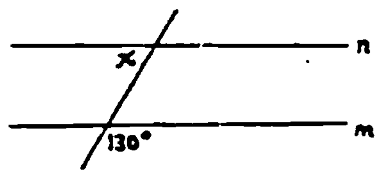


9. The measure of a right angle is

(a) less than 90°
(b) between 90° and 180°
(c) 45°
(d) 90°
(e) 180°

10. Lines n and m are parallel. The measure of angle x is

(a) 65°
(b) 130°
(c) 30°
(d) 40°
(e) 50°

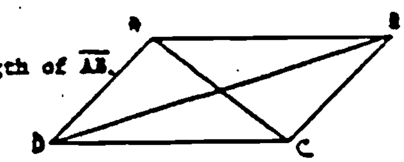


11. An equilateral triangle has

(a) all three sides the same length
(b) one obtuse angle
(c) two angles having the same measure and the third a different measure
(d) all three sides of different lengths
(e) all three angles of different measures

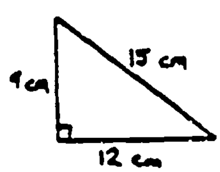
12. Given that ABCD is a parallelogram, which of the following statements is true?

(a) ABCD is equiangular
(b) Triangle ABD is congruent to triangle CDB.
(c) The perimeter of ABCD is four times the length of $\overline{AB}$.
(d) $\overline{AC}$ is the same length as $\overline{BD}$.
(e) All of the above are true.



13. The area of the triangle shown is

(a) 36 sq cm
(b) 54 sq cm
(c) 72 sq cm
(d) 108 sq cm
(e) 1620 sq cm



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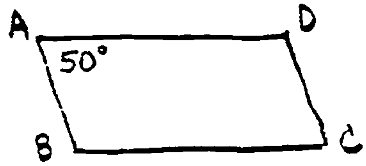
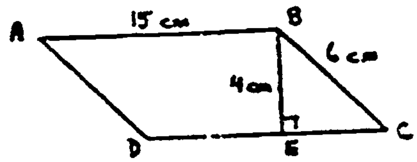
14. ABCD is a parallelogram. The measure of angle C is.

(a) 40°
(b) 130°
(c) 140°
(d) 50°
(e) need more information



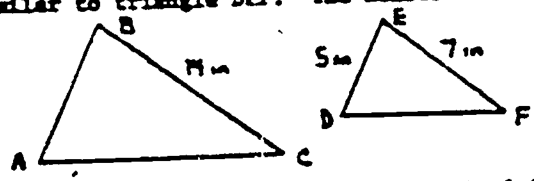
15. The perimeter of this parallelogram ABCD is

(a) 25 cm
(b) 42 cm
(c) 21 cm
(d) 60 cm
(e) 90 cm



16. Triangle ABC is similar to triangle DEF. The measure of $\overline{AB}$ is

(a) 10 in
(b) 11 in
(c) 12 in
(d) 13 in
(e) 15 in

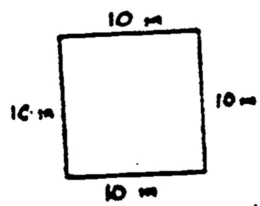


17. The plane figure produced by drawing all points exactly 6 inches from a given point is a

(a) circle with a diameter of 6 inches
(b) square with a side of 6 inches
(c) sphere with a diameter of 6 inches
(d) cylinder 6 inches high and 6 inches wide
(e) circle with a radius of 6 inches

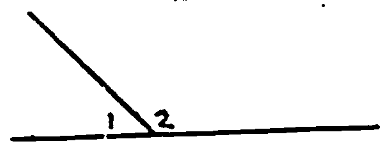
18. The area of the square shown is

(a) 20 sq in
(b) 40 sq in
(c) 40 inches
(d) 100 sq in
(e) 100 inches



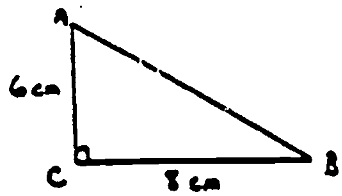
19. Angles 1 and 2 are

(a) interior
(b) vertical
(c) supplementary
(d) complementary
(e) scalene



20. Angle C is a right angle. The length of side $\overline{AB}$ is

(a) 8 cm
(b) 14 cm
(c) 10 cm
(d) 12 cm
(e) 18 cm



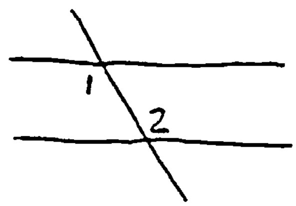
Appendix C

Exiting Geometry Test

1. If two figures are similar but not congruent then they

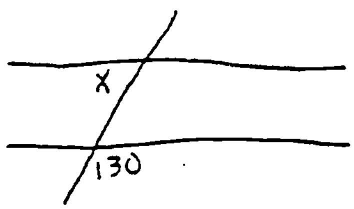
a) have congruent bases and congruent altitudes
b) have the same height
c) both have horizontal bases
d) have a different shape but the same size
e) have a different size but same shape

2. Angles 1 and 2 are called



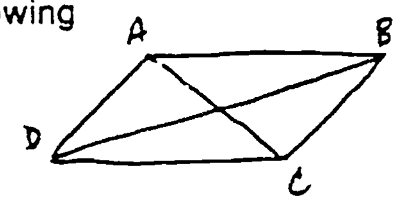
a) opposite angles
b) parallel angles
c) alternate interior angles
d) same side interior angles
e) corresponding angles

3. Lines m and n are parallel. The measure of angle x is



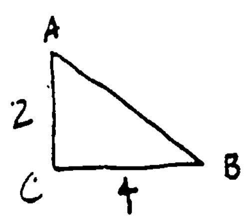
a) 65°
b) 130°
c) 30°
d) 40°
e) 50°

4. Given that ABCD is a parallelogram, which of the following statements is true?



a) ABCD is equiangular
b) triangle ABD is congruent to triangle CDB
c) the perimeter of ABCD is four times the length of AB
d) $\overline{AC}$ is the same length as $\overline{BD}$
e) All of the above are true

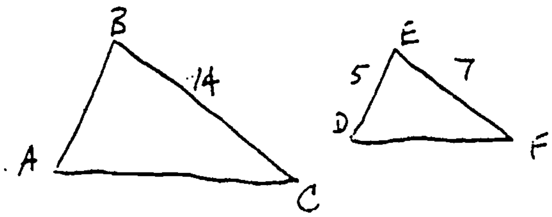
5. Angle C is a right angle. The length of side AB is



a) 20
b) 8
c) $2\sqrt{5}$
d) 16
e) $10\sqrt{2}$

6. Triangle ABC is similar to triangle DEF. The measure of AB is

a) 10 inches
b) 11 inches
c) 12 inches
d) 13 inches
e) 15 inches



7. The supplement of an acute angle is a(n)

a) acute angle
b) complementary angle
c) obtuse angle
d) right angle
e) corresponding

8. A quadrilateral that always has exactly two parallel sides is called

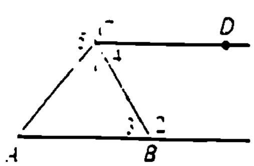
a) a parallelogram
b) a rhombus
c) a rectangle
d) a trapezoid
e) none of the above

9. The sum of the degrees in a regular octagon is

a) 360°
b) 1080°
c) 1440°
d) 135°
e) 720°

10. In the diagram, $\overline{AB} \parallel \overline{CD}$. angle A = 50 degrees, and angle 1 = 70 degrees. The measure of (angle 4 + angle 5) is

a) 110 degrees
b) 120 degrees
c) 170 degrees
d) 180 degrees
e) 210 degrees



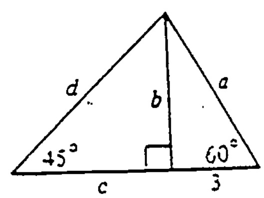
11. Two sides of a triangle are 10 and 13. The third side must be

a) greater than 13 and less than 10
b) greater than 10 and less than 13
c) greater than 3 and less than 13
d) greater than 23 and less than 3
e) greater than 3 and less than 23

12. The measure of "a" is

a) 6
b) $3\sqrt{3}$
c) $\sqrt{3}$
d) 2
e) $6\sqrt{3}$

for 12, 13



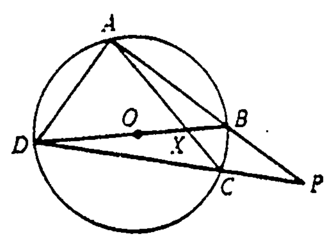
13. The measure of "d" is

a) $3\sqrt{3}$
b) $3\sqrt{6}$
c) $6\sqrt{2}$
d) 6
e) $3\sqrt{2}$

14. In circle O, arc $\overset{\frown}{BC}$ = 20 degrees, and arc $\overset{\frown}{AB}$ = 100 degrees. The measure of arc $\overset{\frown}{DC}$ is

a) 180 degrees
b) 100 degrees
c) 160 degrees
d) 80 degrees
e) none of the above

for 14, 15



15. The measure of angle P is

a) 45 degrees
b) 50 degrees
c) 30 degrees
d) 10 degrees
e) 40 degrees

16. The area of a right triangle with legs of 6 and 3 is

a) 18
b) 12
c) 9
d) $\sqrt{18}$
e) $3\sqrt{2}$

17. The volume of a cylinder with height 10 and base diameter of 10 is

a) 200π
b) 100
c) 250π
d) 350π
e) 250

18. The x-coordinate of the midpoint of the line segment joining the points (7,-4) and (-3,10) is

a) 4
b) 5
c) -4
d) 2
e) -2

19. The slope of a line which is perpendicular to line PQ, where P and Q are the points P(3,4) and Q(-2,-5) is

a) $-\frac{5}{9}$ b) $\frac{9}{5}$ c) $\frac{5}{9}$ d) $-\frac{9}{5}$ e) $\frac{1}{5}$

20. The perimeter of a rhombus with diagonals of 6 and 8 is

a) 28
b) 20
c) 24
d) 48
e) 16

Appendix D

Answer Sheets

Van Hiele Geometry Test
Answer Sheet

name:

grade level: 8 9 10 11 12
(please circle)

age: male/female

current math class: Geometry
(please circle) Algebra

Directions:

Please darken in the letter which corresponds with the appropriate answer.

1.	A	B	C	D	E
2.	A	B	C	D	E
3.	A	B	C	D	E
4.	A	B	C	D	E
5.	A	B	C	D	E
6.	A	B	C	D	E
7.	A	B	C	D	E
8.	A	B	C	D	E
9.	A	B	C	D	E
10.	A	B	C	D	E
11.	A	B	C	D	E
12.	A	B	C	D	E
13.	A	B	C	D	E
14.	A	B	C	D	E
15.	A	B	C	D	E
16.	A	B	C	D	E
17.	A	B	C	D	E
18.	A	B	C	D	E
19.	A	B	C	D	E
20.	A	B	C	D	E
21.	A	B	C	D	E
22.	A	B	C	D	E
23.	A	B	C	D	E
24.	A	B	C	D	E
25.	A	B	C	D	E

Do not write in this box
correlation factors

VHL: $\boxed{}$

1.
2.
3.
4.

Exiting Geometry Test
Answer Sheet

name:

grade level: 8 9 10 11 12
(please circle)

age: male/female

current math class: Geometry
(please circle) Algebra

Directions:

Please darken in the letter which corresponds with the appropriate answer.

1.	A	B	C	D	E
2.	A	B	C	D	E
3.	A	B	C	D	E
4.	A	B	C	D	E
5.	A	B	C	D	E
6.	A	B	C	D	E
7.	A	B	C	D	E
8.	A	B	C	D	E
9.	A	B	C	D	E
10.	A	B	C	D	E
11.	A	B	C	D	E
12.	A	B	C	D	E
13.	A	B	C	D	E
14.	A	B	C	D	E
15.	A	B	C	D	E
16.	A	B	C	D	E
17.	A	B	C	D	E
18.	A	B	C	D	E
19.	A	B	C	D	E
20.	A	B	C	D	E

RAW SCORE

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